Assimilation de données pour la reconstruction d'écoulements

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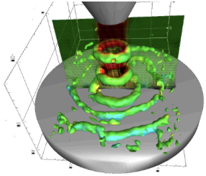
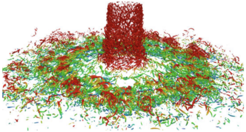
Confronting EFD and CFD is inherent of fluid mechanics approaches

TomoPIV (Irstea)
Re 2 500



DNS (Dairy *et al.*,2015)
Re 10 000



Experiments

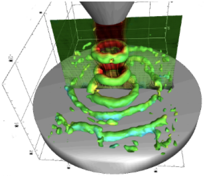
- LDV as a reference
- HWA $\rightarrow$ very good
- PIV $\rightarrow$ good

Numerical simulations

- DNS as a reference
  $\rightarrow$ numerical wind tunnel
- A priori parameter calibration
- A posteriori simulation validation

EFD and CFD limitations

TomoPIV (Irstea)
Re 2 500

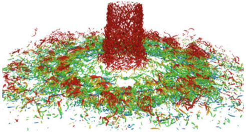


Experiments

- HWA and LDV $\rightarrow$ pointwise
- PIV $\rightarrow$ large scale
- TomoPIV $\rightarrow$ very large scale

$\Rightarrow$ sparse data

DNS (Dairy *et al.*,2015)
Re 10 000



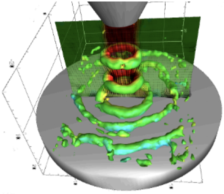
Numerical simulations

- Initial conditions
- Boundary conditions
- Turbulence model and parameters

$\Rightarrow$ non "realistic" simulations

Coupling EFD and CFD with data assimilation

TomoPIV (Irstea)
Re 2 500



DNS (Dairy *et al.*,2015)
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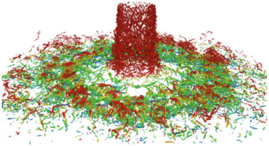


Objective

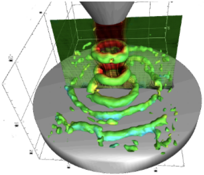
- Estimation of the unknown true state of interest $\mathbf{x}(t, x)$
- Recover as accurately as possible the state of the fluid flow using all available information

Question: how to do that ?

Outline

Data assimilation ingredients

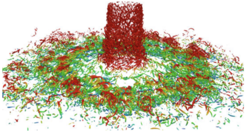
TomoPIV (Irstea)



Experiments

- Observation model
  $\boldsymbol{\mathcal{Y}}(t,x) = \mathbb{H}(\mathbf{x}(t,x)) + \boldsymbol{\varepsilon}(t,x)$

DNS (Dairy *et al.*,2015)

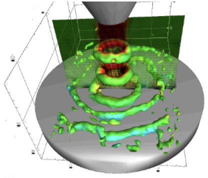


Numerical model

- Dynamical model
  $\partial_t \mathbf{x}(t,x) + \mathbb{M}(\mathbf{x}(t,x)) = \mathbf{q}(t,x)$
- Prior knowledge model
  $\mathbf{x}(t_0,x) = \mathbf{x}_0^b + \boldsymbol{\eta}(x)$

Data and dynamics dimensions

TomoPIV (Irstea)



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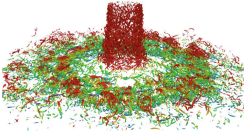
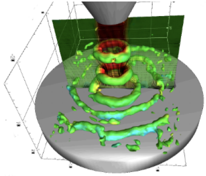


Data and model resolution: d vs m

- Geosciences $d << m$
- PIV $d \leq m$ or $d << m$
  - Model resolution: ROM vs DNS
  - Laboratory vs Industrial processes
  - 2D vs 3D
  - Reynolds

Data assimilation: observation and dynamics models

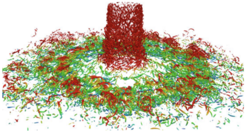
TomoPIV (Irstea)



$$\mathcal{Y}(t,x) = \mathbb{H}(\mathbf{x}(t,x)) + \varepsilon(t,x)$$

- Pseudo observation → velocity, vorticity, lagrangian acceleration, thus $\mathcal{Y}(t,x) = \hat{\mathbf{x}}(t,x)$ and $\mathbb{H} = \mathbb{I}$
- Observation → images of particles, scalar (smoke, gaz, temperature), thus $\mathcal{Y}(t,x) = I(t,x)$ and $\mathbb{H}$ can be nonlinear
- Eulerian or Lagrangian obs.

DNS (Dairy *et al.*,2015)

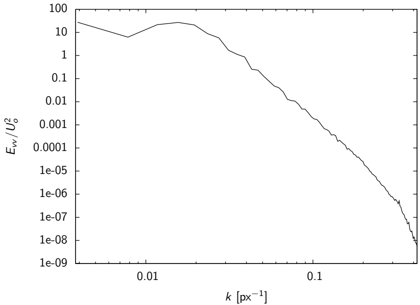


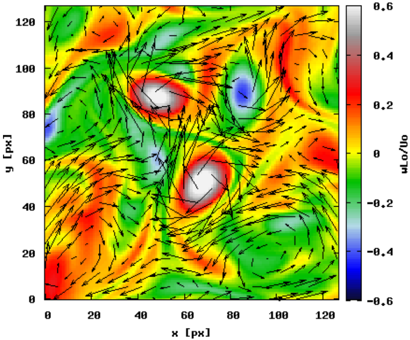
$$\partial_t \mathbf{x}(t,x) + \mathbb{M}(\mathbf{x}(t,x)) = \mathbf{q}(t,x)$$

- Eulerian: ROM, Vortex particle, Lattice Boltzman, RANS, LES, DNS
- Lagrangian: Smooth Particule Hydrodynamics (SPH)

Data assimilation ideal case

Papadakis & Mémin (2008) - Heitz *et al.* (2010)





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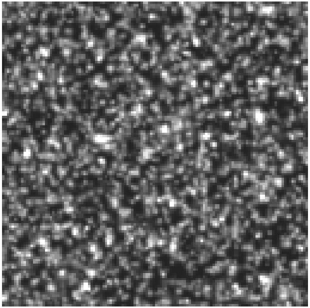
- Pseudo observation $\rightarrow$ velocity, thus $\mathcal{Y}(t,x) = \hat{\mathbf{x}}(t,x)$ and $\mathbb{H} = \mathbb{I}$

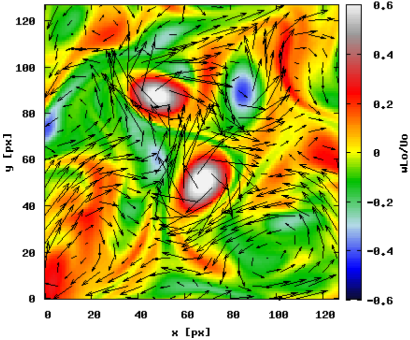
$$\partial_t \mathbf{x}(t,x) + \mathbb{M}(\mathbf{x}(t,x)) = 0$$

- DNS of 2D IHT at $Re = 256$
- Resolution : 256×256

Data assimilation ideal case

Papadakis & Mémin (2008) - Heitz *et al.* (2010)





$$\partial_t I(t,x) + \mathbf{x} \cdot \nabla I(t,x) = \varepsilon(t,x)$$

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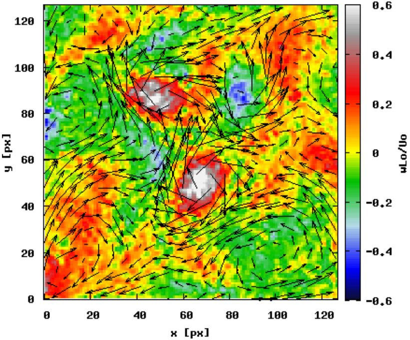

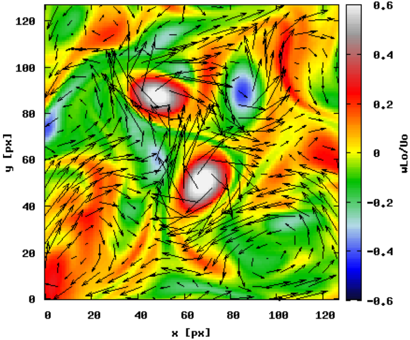

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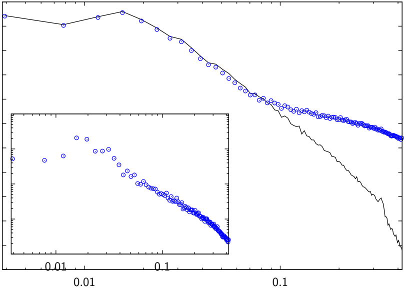

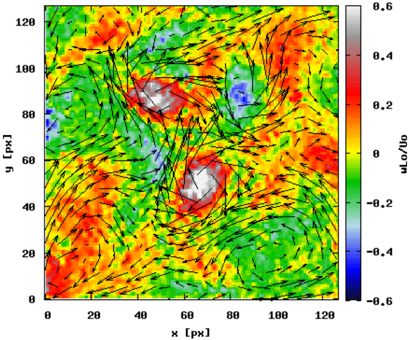

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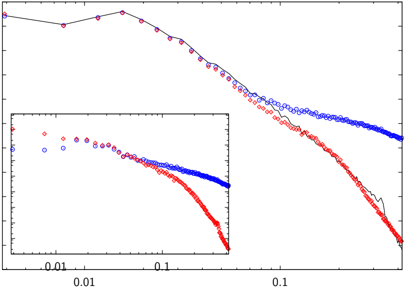

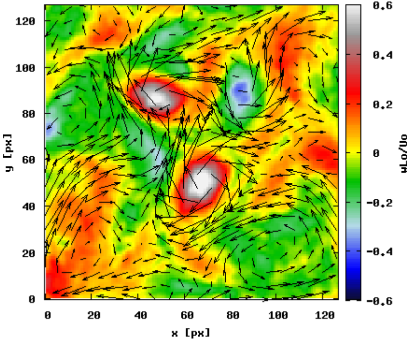

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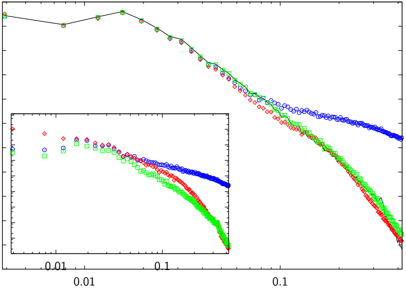

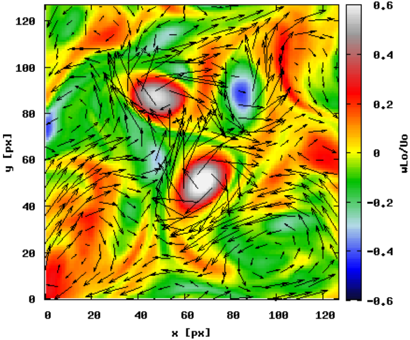

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Outline

Data assimilation: the state estimation problem

Ingredients

- Observation model
  $\mathcal{Y}(t,x) = \mathbb{H}(\mathbf{x}(t,x)) + \varepsilon(t,x)$
- Dynamical model
  $\partial_t \mathbf{x}(t,x) + \mathbb{M}(\mathbf{x}(t,x)) = \mathbf{q}(t,x)$
- Prior knowledge model
  $\mathbf{x}(t_0,x) = \mathbf{x}_0^b + \boldsymbol{\eta}(x)$

→ Random nature of observation, dynamic and knowledge errors described in term of pdf

Bayesian formulation

$$p(\mathbf{x}|\mathcal{Y}) = \frac{p(\mathcal{Y}|\mathbf{x})p(\mathbf{x})}{p(\mathcal{Y})}$$

$$p(\mathbf{x}|\mathcal{Y}) \propto p(\mathcal{Y}|\mathbf{x})p(\mathbf{x})$$

posterior ∝ *likelihood* × *prior*

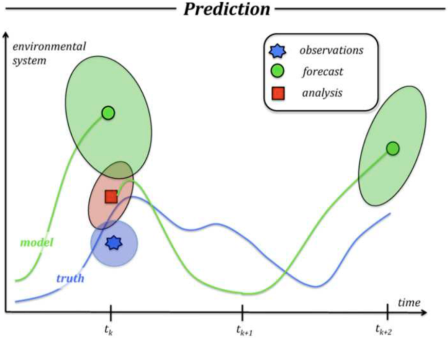
analysis ∝ *observations* × *knowledge*

Prior distribution:

- Prevents from over-fitting
- Introduce past information
- Good prior not straightforward

Data assimilation: the state estimation problem

Carassi *et al.* (2017)



Information: past

$\rightarrow$ For the control?

Bayesian formulation

$$p(\mathbf{x}|\mathcal{Y}) \propto p(\mathcal{Y}|\mathbf{x})p(\mathbf{x})$$

analysis $\propto$ *observations* $\times$ *knowledge*

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Data assimilation: the state estimation problem

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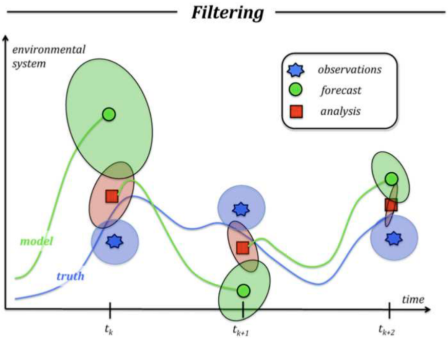

Information: past and present

→ Sequential processing providing discontinuous trajectories

Bayesian formulation

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analysis $\propto$ *observations* $\times$ *knowledge*

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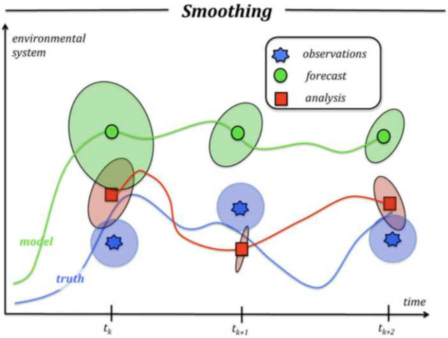

Information: past, present and future

→ Relevant for reconstruction or reanalysis and for model parameters estimation

Bayesian formulation

$$p(\mathbf{x}|\mathcal{Y}) \propto p(\mathcal{Y}|\mathbf{x})p(\mathbf{x})$$

analysis $\propto$ *observations* $\times$ *knowledge*

Prior distribution:

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Data assimilation: the state estimation problem

Computational problem

- Huge dimension of data and models prevent use of fully Bayesian approach
- Difficulty to define and transport the pdfs

Solution to overcome this issue

- Uncertainties of observations, model and prior are assumed Gaussian
- Pdfs completely described by first and second moments (i.e mean and covariance matrix)

Bayesian formulation

$$p(\mathbf{x}|\mathcal{Y}) \propto p(\mathcal{Y}|\mathbf{x})p(\mathbf{x})$$

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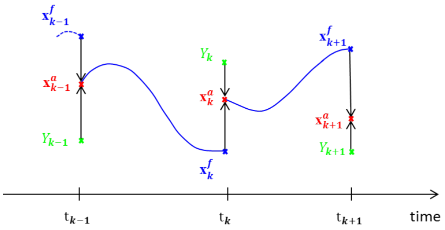
Sequential data assimilation: Kalman filter



Properties

- Obs. and dynamics linear
- Noises Gaussian, unbiased, white-in-time

→ Time dependent prior (mean, cov.)

→ Comput. cost. of K and P

Main algorithm

1. Forecast step

$$\mathbf{x}^{\mathrm{f}}_k = \mathbf{M}_{k:k-1}\mathbf{x}^{\mathrm{a}}_{k-1},$$

$$\mathbf{P}^{\mathrm{f}}_k = \mathbf{M}_{k:k-1}\mathbf{P}^{\mathrm{a}}_{k-1}\mathbf{M}^{\mathrm{T}}_{k:k-1} + \mathbf{Q}_k.$$

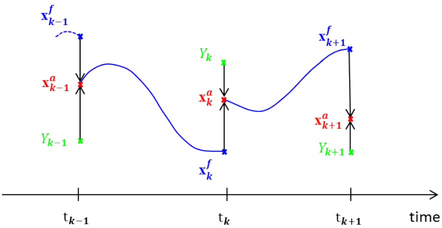
2. Analysis step

$$\mathbf{K}_k = \mathbf{P}^{\mathrm{f}}_k\mathbf{H}^{\mathrm{T}}_k(\mathbf{H}_k\mathbf{P}^{\mathrm{f}}_k\mathbf{H}^{\mathrm{T}}_k + \mathbf{R}_k)^{-1},$$

$$\mathbf{x}^{\mathrm{a}}_k = \mathbf{x}^{\mathrm{f}}_k + \mathbf{K}_k(\mathbf{y}_k - \mathbf{H}_k\mathbf{x}^{\mathrm{f}}_k),$$

$$\mathbf{P}^{\mathrm{a}}_k = (\mathbf{I}_k - \mathbf{K}_k\mathbf{H}_k)\mathbf{P}^{\mathrm{f}}_k.$$

Sequential data assimilation: Kalman filter



Properties

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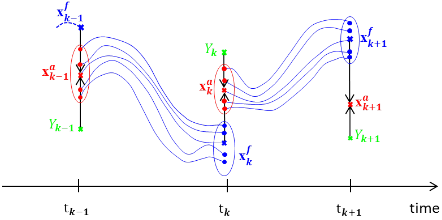
Alternative approaches

- Extended Kalman Filter (EKF)

  → H and M linearized
- Sub Optimal Filter (SOS)

  → Reduce comput. cost H

Sequential data assimilation: Kalman filter



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→ Time dependent prior (mean, cov.)

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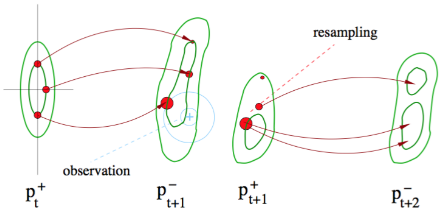
Alternative approaches

- Extended Kalman Filter (EKF)
  → H and M linearized
- Sub Optimal Filter (SOS)
  → Reduce comput. cost H
- Ensemble Kalman Filter (EnKF)
  → Empirical estimation of P
  → H and M non linear

Sequential data assimilation: Kalman filter



From Boquet's lecture notes (2014-2015)

Properties

- Obs. and dynamics linear
- Noises Gaussian, unbiased, white-in-time

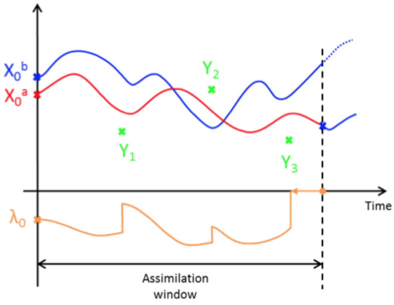
→ Time dependent prior (mean, cov.)

→ Comput. cost. of K and P

Alternative approaches

- Extended Kalman Filter (EKF)
  → H and M linearized
- Sub Optimal Filter (SOS)
  → Reduce comput. cost H
- Particle Filter (PF)
  → H and M non linear
  → Noises: non-Gaussian, biased, multimodal
  → Sampling issues due to high dimensions

Variational data assimilation: Classical 4DVar



Energy function

$$J(\mathbf{x}_0) = \tfrac{1}{2}\|\mathbf{x}_0 - \mathbf{x}_0^b\|_B^2 + \frac{1}{2}\int_{t_0}^{t_f} \|\mathbb{H}(\mathbf{x}) - \boldsymbol{\mathcal{Y}}\|_R^2 dt,$$

$$\text{s.t.} \quad \partial_t \mathbf{x}(t,x) + \mathbb{M}(\mathbf{x}(t,x),u) = 0.$$

- Computing the gradient of $J(\mathbf{x}_0)$ is very expensive!
- Deduced by solving the backwards adjoint equation

$$-\partial_t \boldsymbol{\lambda}(t) + (\partial_{\mathbf{x}}\mathbb{M})^* \boldsymbol{\lambda}(t) = (\partial_{\mathbf{x}}\mathbb{H})^* \mathbf{R}^{-1}(\boldsymbol{\mathcal{Y}}(t) - \mathbb{H}($$

$$\boldsymbol{\lambda}(t_f) = 0$$

Properties

- Obs. and dynamics non-linear
- Noises Gaussian, unbiased, white-in-time

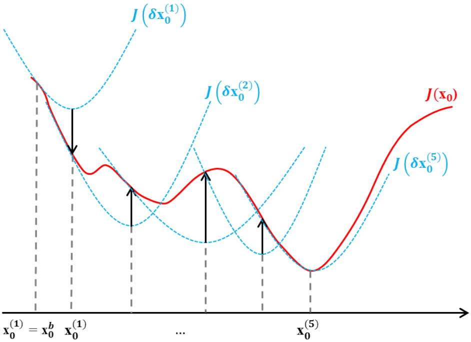
$\rightarrow$ Time independent prior (B)

$\rightarrow$ Derivation of the adjoint model

Variational data assimilation: Incremental 4DVar

Objective

Avoid local minima by solving a convex optimization problem under the constraint of the linearized dynamical model.



New minimization problem formulated by the convex cost function

$$J(\delta \mathbf{x}_0^{(i)}) = \frac{1}{2}\|\delta \mathbf{x}_0^{(i)} + \mathbf{x}_0^{(i)} - \mathbf{x}_0^b\|^2_{\mathbf{B}^{-1}} + \frac{1}{2}\int_{t_0}^{t_f} \|\mathbf{H}\delta \mathbf{x}^{(i)}(t) + \mathbb{H}(\mathbf{x}_t^{(i)}) - \mathcal{Y}(t)\|^2_{\mathbf{R}^{-1}} dt$$

under the constraint of the linearized dynamics equations

$$\partial_t \delta \mathbf{x}^{(i)} + \partial_{\mathbf{x}} \mathbb{M}(\mathbf{x}^{(i)}).\delta \mathbf{x}^{(i)} = 0$$

$$\delta \mathbf{x}_0^{(i)} = \mathbf{x}_0^b - \mathbf{x}_0^{(i)}$$

Variational data assimilation: Incremental 4DVar

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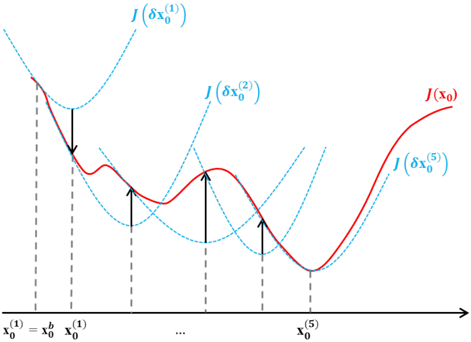
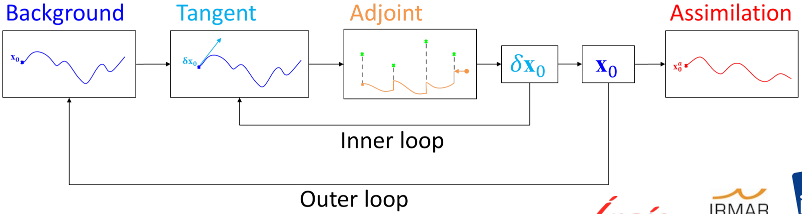
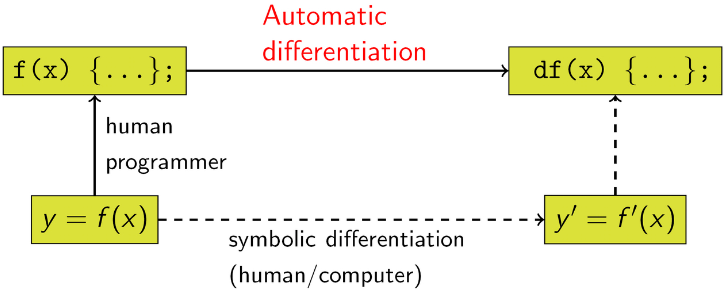




Variational data assimilation: 4DVar adjoint construction



Variational data assimilation: 4DVar adjoint construction

- Nonlinear dynamics

$$x_0 \xrightarrow{I_1} \dots \xrightarrow{I_j} x_j = I_j(x_{j-1}) \xrightarrow{I_{j+1}} \dots \xrightarrow{I_p} x_p$$

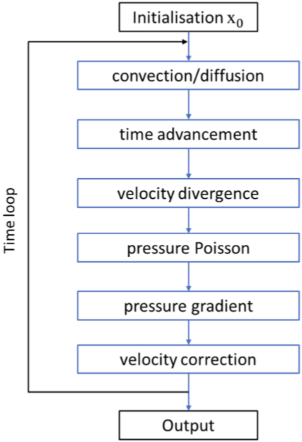
- Tangent procedure

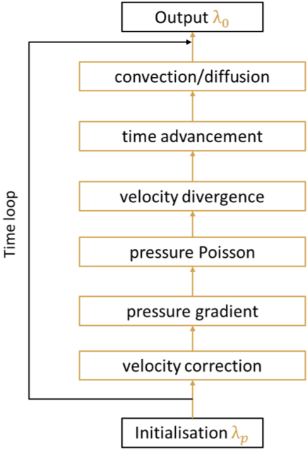
$$\delta x_0 \xrightarrow{I'_1} \dots \xrightarrow{I'_j} \delta x_j = I'_j(x_{j-1}).\,\delta x_{j-1} \xrightarrow{I'_{j+1}} \dots \xrightarrow{I'_p} \delta x_p$$

- Adjoint procedure

$$\lambda_0 \xleftarrow{I^*_1} \dots \xleftarrow{I^*_j} \lambda_j = I^*_j(x_{j-1}).\,\lambda_{j-1} \xleftarrow{I^*_{j+1}} \dots \xleftarrow{I^*_p} \lambda_p$$

Variational data assimilation: 4DVar adjoint construction





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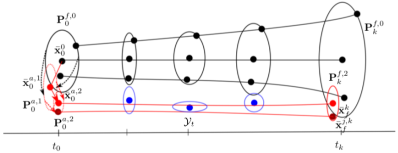
Variational data assimilation: Ensemble Variationnal EnVar



Energy function

$$J(\mathbf{x}_0) = \frac{1}{2}\|\mathbf{x}_0 - \mathbf{x}_0^b\|_B^2 + \frac{1}{2}\int_{t_0}^{t_f} \|\mathbb{H}(\mathbf{x}) - \boldsymbol{\mathcal{Y}}\|_R^2 dt,$$

$$\text{s.t.} \quad \partial_t \mathbf{x}(t,x) + \mathbb{M}(\mathbf{x}(t,x),u) = 0.$$

- Change cost function in terms of weighting vector
- Propagation of $B^{\frac{1}{2}}$ projected into observation space

→ Based on optimization theory

→ Fast operational implementation

→ Uncertainty sample-based or from optimization procedure

→ Localization and inflation

Properties

- Obs. and dynamics non-linear
- Noises Gaussian, unbiased, white-in-time

→ Sample based covariance (B)

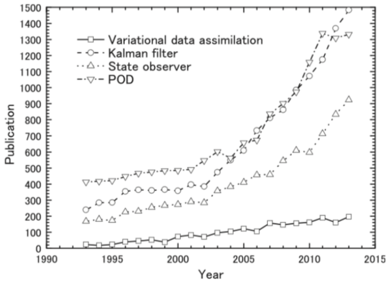
→ Time dependent prior (B)

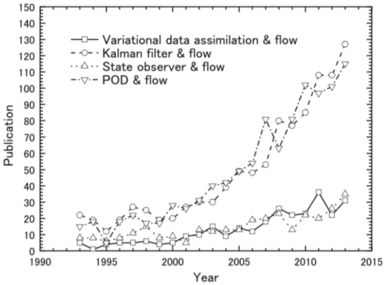
→ No derivation of the adjoint model

Outline

Data assimilation publications: all fields vs fluid flow

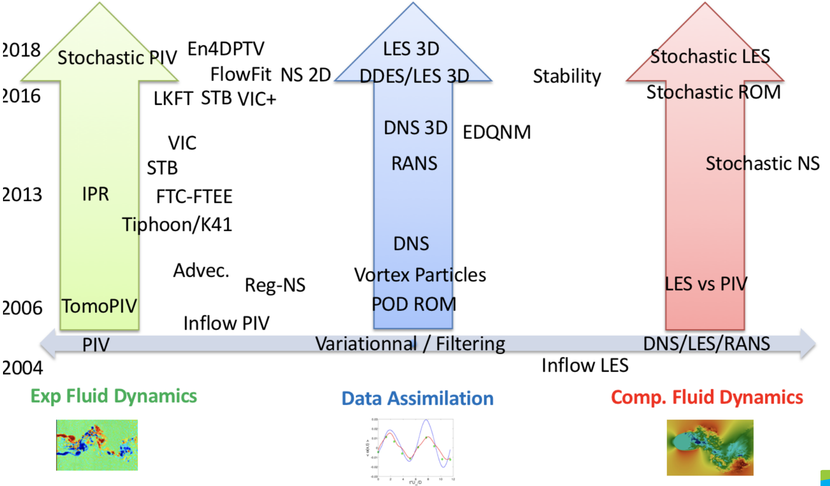
Hayase (2015, Fluid Dyn. Res.)





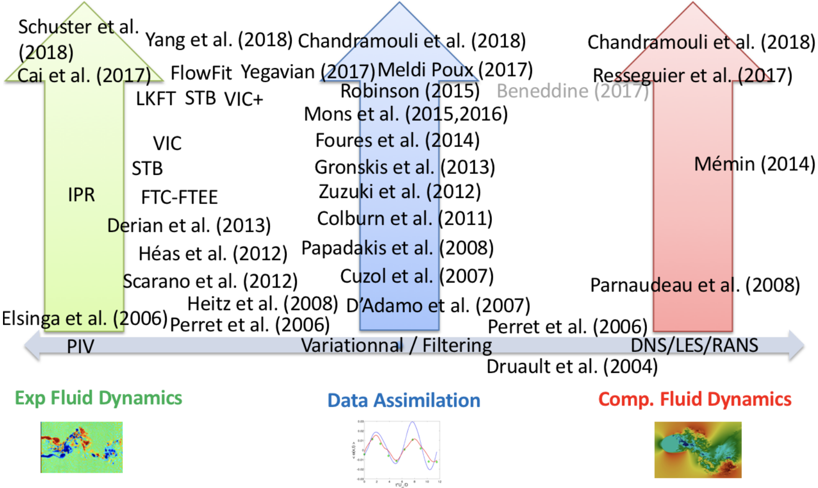
Data assimilation: data-driven vs model-driven

Different modelling



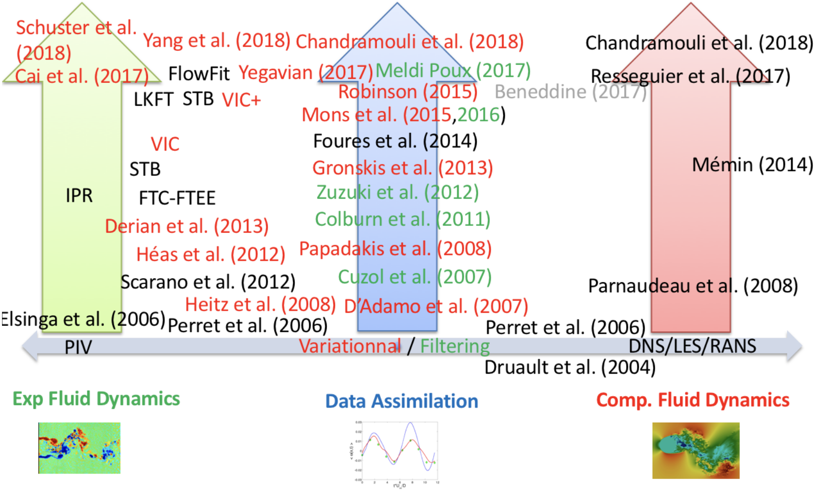
Data assimilation: data-driven vs model-driven

Non exhaustive state of the art



Data assimilation: data-driven vs model-driven

Variational vs Filtering approaches



Data assimilation: data-driven vs model-driven

3D vs 2D approaches

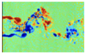
Schuster et al. (2018)
Cai et al. (2017)
Yang et al. (2018)
FlowFit
LKFT STB VIC+
VIC
STB
IPR
FTC-FTEE
Derian et al. (2013)
Héas et al. (2012)
Scarano et al. (2012)
Heitz et al. (2008)
Elsinga et al. (2006)
Perret et al. (2006)

Chandramouli et al. (2018)
Yegavian (2017) Meldi Poux (2017)
Robinson (2015)
Beneddine (2017)
Mons et al. (2015,2016)
Foures et al. (2014)
Gronskis et al. (2013)
Zuzuki et al. (2012)
Colburn et al. (2011)
Papadakis et al. (2008)
Cuzol et al. (2007)
D'Adamo et al. (2007)

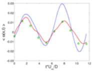
Chandramouli et al. (2018)
Resseguier et al. (2017)
Mémin (2014)
Parnaudeau et al. (2008)
Perret et al. (2006)
Druault et al. (2004)

PIV — Variationnal / Filtering — DNS/LES/RANS

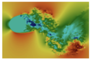
Exp Fluid Dynamics



Data Assimilation



Comp. Fluid Dynamics



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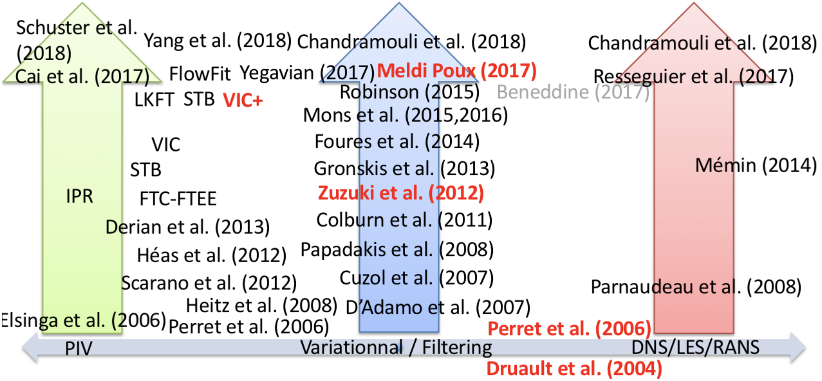
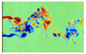
Data assimilation: data-driven vs model-driven

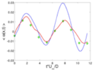
Focus



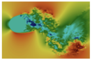
Exp Fluid Dynamics



Data Assimilation



Comp. Fluid Dynamics



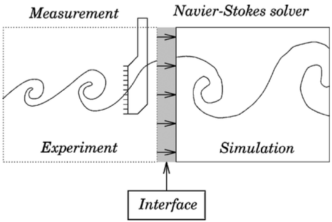
Data assimilation: data-driven vs model-driven

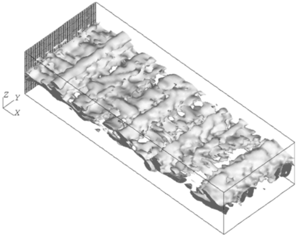
AIAA Journal
Vol. 42, No. 3, March 2004

Generation of Three-Dimensional Turbulent Inlet Conditions for Large-Eddy Simulation

P. Druault*
Université Pierre-et-Marie-Curie, 78210 Saint Cyr l'Ecole, France
S. Lardeau†
Imperial College of Science, Technology, and Medicine, London, England SW7 2BY, United Kingdom
and
J.-P. Bonnet,‡ F. Coiffet,§ J. Delville,¶ E. Lamballais,** J. F. Largeau,§ and L. Perret§
Université de Poitiers, 86962 Futuroscope Chasseneuil CEDEX, France

A method for generating realistic (i.e., reproducing in space and time the large-scale coherence of the flows)





Data assimilation: data-driven vs model-driven

PHYSICS OF FLUIDS **20**, 075107 (2008)

Turbulent inflow conditions for large-eddy simulation based on low-order empirical model

Laurent Perret,[1,a)] Joël Delville,[2] Rémi Manceau,[2] and Jean-Paul Bonnet[2]
[1]*Laboratoire de Mécanique des Fluides (LMF), UMR CNRS 6598, Ecole Centrale de Nantes, 1 rue de la Noë BP 92101, F-44321 Nantes Cedex 3, France*
[2]*Laboratoire d'Etudes Aérodynamiques (LEA), ENSMA, CNRS, CEAT, Université de Poitiers, 43, route de l'aérodrome, F-86036 Poitiers, France*

(Received 30 October 2007; accepted 3 June 2008; published online 22 July 2008)

7)

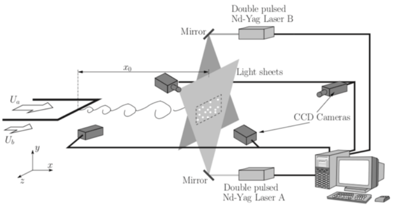
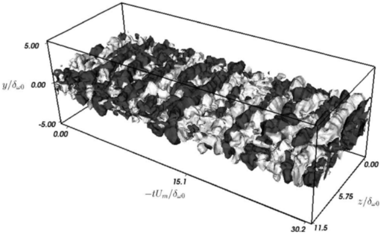
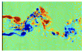
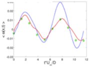
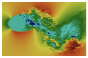


FIG. 1. DT-SPIV setup.









Data assimilation: data-driven vs model-driven

IOP Publishing | The Japan Society of Fluid Mechanics — Fluid *Dynamics* Research

Fluid Dyn. Res. **47** (2015) 051407 (22pp) — doi:10.1088/0169-5983/47/5/051407

Hierarchy of hybrid unsteady-flow simulations integrating time-resolved PTV with DNS and their data-assimilation capabilities

Takao Suzuki[1] and Fujio Yamamoto[2]

$$\hat{\mathbf{u}}_{n+1}^{\mathrm{Hyb}} = \mathcal{N}\left(\mathbf{u}_n^{\mathrm{Hyb}}\right),$$

$$\tilde{\mathbf{u}}_{n+1}^{\mathrm{Hyb}} = \hat{\mathbf{u}}_{n+1}^{\mathrm{Hyb}} + \mathbf{K}\left(\mathbf{u}_{n+1}^{\mathrm{PIV}} - \hat{\mathbf{u}}_{n+1}^{\mathrm{Hyb}}\right),$$

$$\left(\mathbf{u}_{n+1}^{\mathrm{Hyb}} = \mathcal{P}\left(\tilde{\mathbf{u}}_{n+1}^{\mathrm{Hyb}}\right)\right),$$

Types of algorithms
POD-Galerkin (Suzuki 2014)
Conventional (Suzuki *et al* 2010)
Kalman filter (Suzuki 2012)

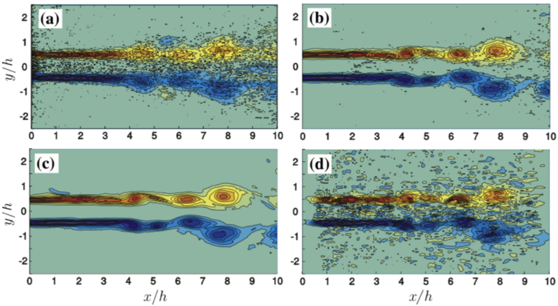


Figure 5. Comparison of vorticity contours among the three hybrid algorithms at the same instant ($tu_{\mathrm{jet}}/h = 11.9$). The jet is ejected from left to right. (a) POD–Galerkin projection. (b) Conventional hybrid algorithm. (c) Kalman-filtered algorithm. (d) Raw PTV. Contour levels: $-5.1 \leqslant \omega \leqslant 5.1$ with a $\Delta\omega = 0.6$ increment for all.

Data assimilation: data-driven vs model-driven

Journal of Computational Physics 347 (2017) 207–234

Contents lists available at ScienceDirect

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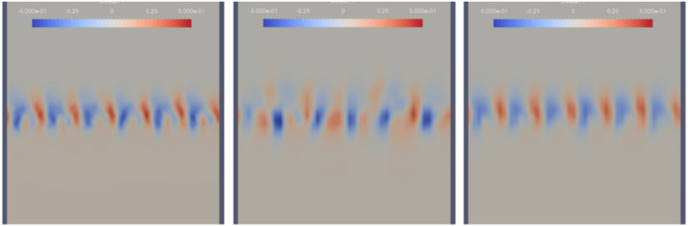
A reduced order model based on Kalman filtering for sequential data assimilation of turbulent flows

M. Meldi *, A. Poux [1]



(a) (b) (c)

Fig. 17. Isocontours of the time averaged normal velocity $\overline{u_y}$ taken at the streamwise section $x = 16\Lambda$. A zoom around the wake region is performed. Results for (a) the DNS calculation, (b) the LES simulation and (c) the *observer* estimator are shown, respectively.

Data assimilation: data-driven vs model-driven

Exp Fluids (2016) 57:139
DOI 10.1007/s00348-016-2225-6

CrossMark

RESEARCH ARTICLE

Dense velocity reconstruction from tomographic PTV with material derivatives

Jan F. G. Schneiders[1] · Fulvio Scarano[1]

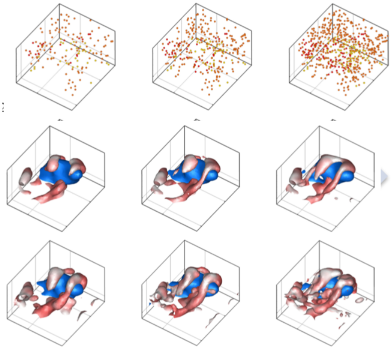
$$J = J_u + \alpha^2 J_{Du}, \quad (6)$$

where α is a weighting coefficient (Sect. 2.3.3), J_u is given by Eq. (7) and J_{Du} is given by Eq. (8),

$$J_u = \sum_p \left\| u_h(x_p) - u_m(x_p) \right\|^2, \quad (7)$$

$$J_{Du} = \sum_p \left\| \frac{Du_h}{Dt}(x_p) - \frac{Du_m}{Dt}(x_p) \right\|^2, \quad (8)$$

where u_h and Du_h/Dt are calculated from Eqs. (1) and (2) and are evaluated at the particle locations, x_p, by linear interpolation from the computational grid. The cost function penalizes the difference between the PTV measurements and the velocity and material derivative at a single measurement time-instant calculated from the optimization variables. The optimization problem does not include time-integration of the vorticity transport equation.



Outline

Data assimilation ingredients

Data assimilation tools

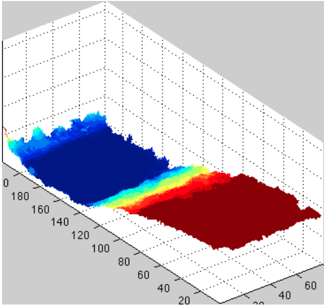
Overview of significant achievements

Some applications

- Sequential assimilation
- Variational assimilation

Wave in a rectangular flat bottom tank

Depth observations



Dynamical model

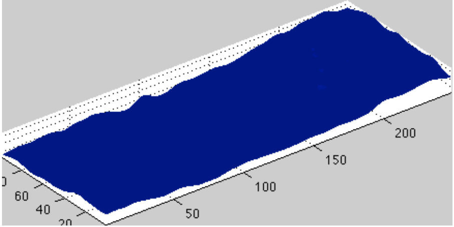
- **Shallow water model**

$$\partial_t H + \nabla \cdot (H\mathbf{v}) = 0.$$
$$\partial_t (H\mathbf{v}) + \nabla \cdot (H\mathbf{v}\mathbf{v}) = -gH\eta + F.$$

Data assimilation approaches

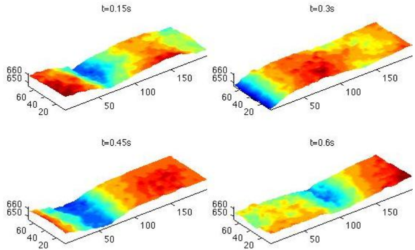
- **WEnKF** (`Combès` *et al.*, `2015, Fluid Dyn Res`)
- Error model (obs. and dynamics)

Data assimilation results



- Reconstruct unobserved surface velocity

Wave in a rectangular flat bottom tank



Flow configuration

- $Lx \times Ly = 250$ mm $\times$ 100 mm
- Initial free surface height difference $h_0 = 1$ cm
- Observations every $10\Delta t\, u_0/L_x$ leading to $St_{\mathrm{obs}} \simeq 24$, that was rather high !

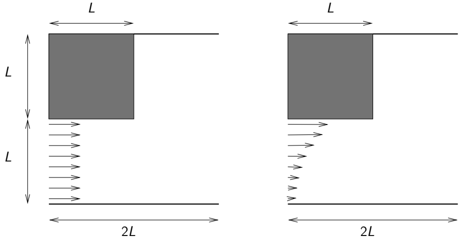
Simulation parameters

- Shallow water model
- $n_x \times n_y = 222 \times 88$
- $\Delta t\, u_0/L_x = 0.0042$

Assimilation parameters

- particle number $N = 100$
- $\mathbf{x}_{\mathrm{init}} = (0, 0, 0)$
- $\mathbf{X}_0 \sim \mathcal{N}(\mathbf{x}_{\mathrm{init}}, \mathbf{R}_0)$
- $\mathbf{W}_t^f \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_t)$
- $\mathbf{W}_t^g \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_t)$
- $\mathbf{R}_0$ $(0.05\, h_0; 0.25\, u_0; r_h)$
- $\mathbf{R}_t$ $(0.04\, h_0; 0.06\, u_0; r_h)$
- $\mathbf{Q}_t$ $(0.013\, h_0^2; diag.)$
- localization $h_{correl} = 0.6 h_0$

Suddenly expanding flume



Flow configurations

- $L = 10$ cm
- Inflow velocity and elevation oscillatory in phase at 1 Hz with $H_{\rm in} = 1$ cm and $V_{\rm in} = 0.22$ m/s
- $Fr = U_{\rm in}/\sqrt{g\,H_{\rm in}} = 0.7$

Simulation parameters

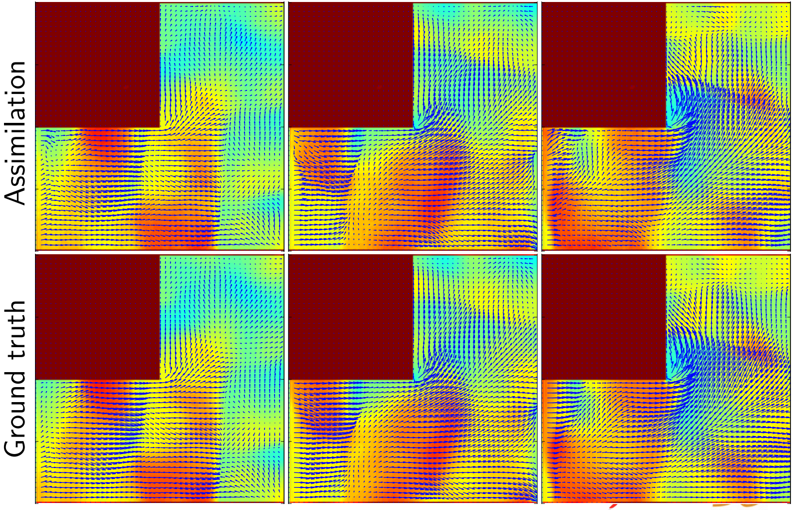
- Shallow water model
- $n_x \times n_y = 200 \times 200$
- $\Delta t\, u_0/L = 0.006$

Assimilation parameters

- particle number $N = 100$
- $\mathbf{x}_{\rm init} = (0, 0, 0)$
- $\mathbf{X}_0 \sim \mathcal{N}(\mathbf{x}_{\rm init}, \mathbf{R}_0)$
- $\mathbf{W}_t^f \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_t)$
- $\mathbf{W}_t^g \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_t)$
- $\mathbf{R}_0$ $(0.05\, h_0; 0.25\, u_0; r_h)$
- $\mathbf{R}_t$ $(0.04\, h_0; 0.06\, u_0; r_h)$
- $\mathbf{Q}_t$ $(0.013\, h_0^2; diag.)$
- localization $h_{correl} = 0.6 h_0$

Suddenly expanding flume

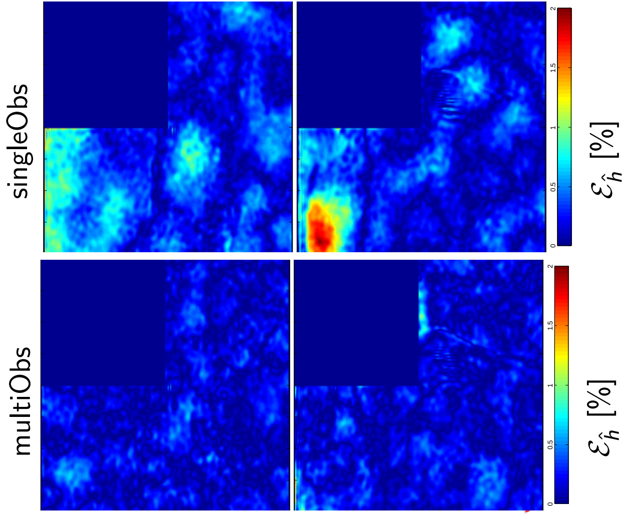
Non-uniform inlet velocity profile (with spatial complexity)



$t\, u_0/L = 1.19$ $t\, u_0/L = 1.57$ $t\, u_0/L = 1.98$

Suddenly expanding flume

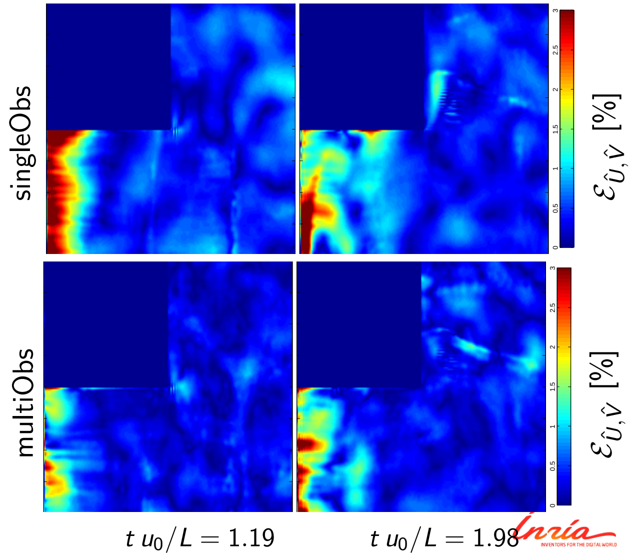
Elevation error maps for singleObs and multiObs



$t\,u_0/L = 1.19$ $t\,u_0/L = 1.98$

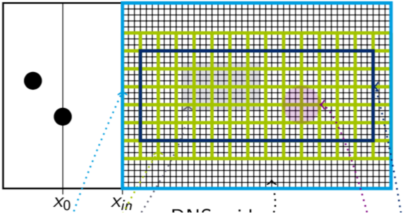
Suddenly expanding flume

Velocity error maps for singleObs and multiObs



Cylinder wakes at *Re*=112

PIV observations



Dynamical model

- **DNS with code Incompact3d** (Laizet et al., 2010 JCP)

Data assimilation approaches

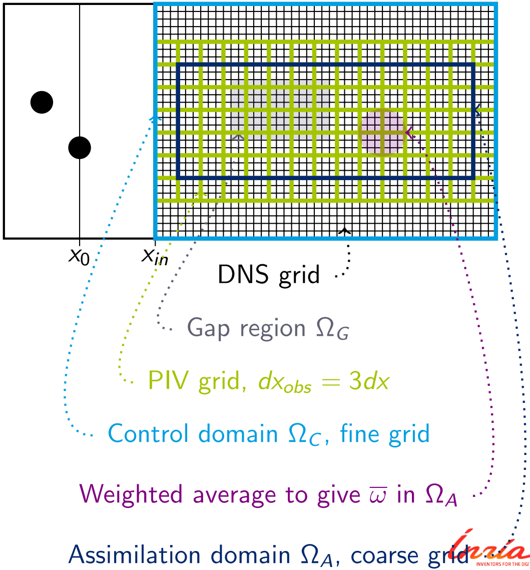
- **Classical 4DVar approach** from `Gronskis` *et al.* (2013)
- Inflow and initial condition control

Data assimilation results

- Reconstruct inflow and initial condition
- Reconstruct gap
- Influence of gap size & obs. frequency
- Reconstruct pressure, Lift & Drag

Cylinder wakes at Re=112

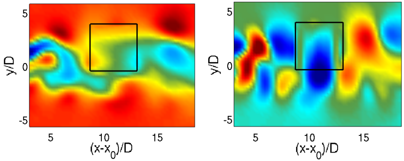
4DVar approach from Gronskis *et al.*(2013)



Cylinder wakes at $Re=112$

Initial condition

$u_x^{obs}(\mathbf{x}, t_0)$ $u_y^{obs}(\mathbf{x}, t_0)$

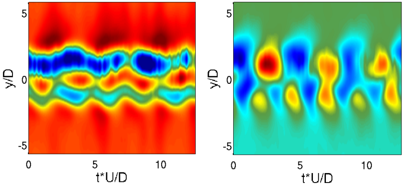


Initial condition in Ω_G (**IC**)

1. Uniform stagnant flow
2. Velocity interpolation

Inflow condition

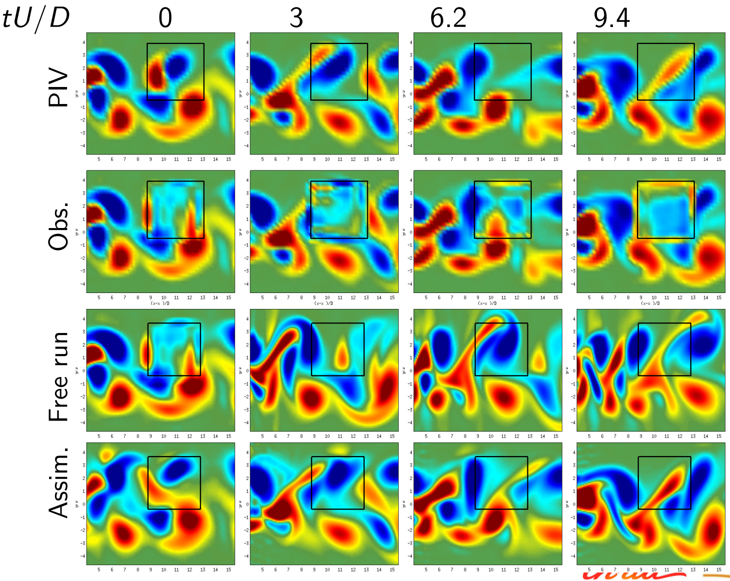
$u_x^{k=0}(\mathbf{x}_{in}, t)$ $u_y^{k=0}(\mathbf{x}_{in}, t)$



- From PIV sequence with Taylor's hypothesis

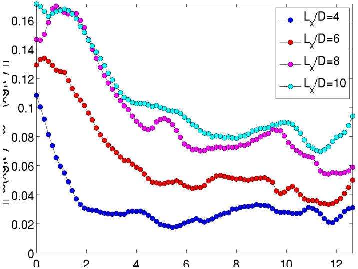
Cylinder wakes at *Re*=112

Gap reconstruction



Cylinder wakes at Re=112

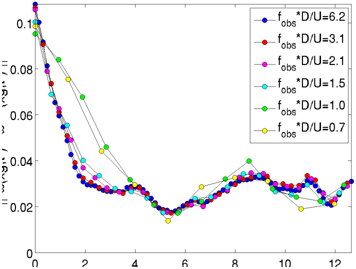
Influence of gap size



$IC=1,\ f_{obs}D/U=6.2$

Method's accuracy was strongly related to the size of the gap.

Influence of obs. frequency



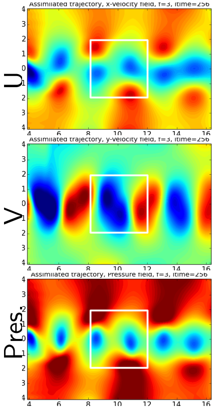
$L_x/D=4,\ IC=1$

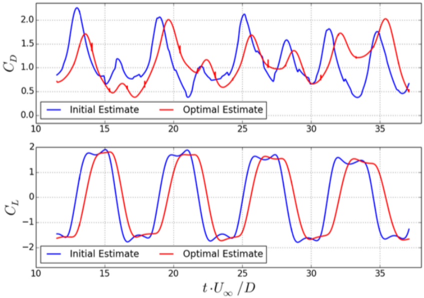
Error decreased with increasing observations frequency $St_{obs} = f_{obs}\,D/U$.

Cylinder wakes at Re=112

Pressure, Drag and Lift reconstruction via 4DVar

Gronskis *et al.*(2018, CFTL)

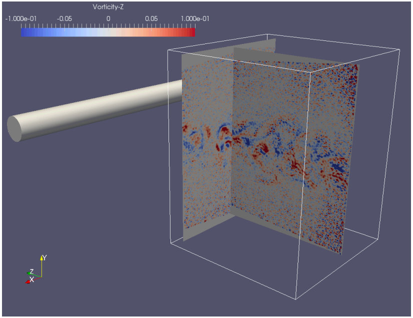




- Reconstruct unobserved pressure
- Lift and Drag via control volume

Cylinder wake at Re=3900

Cross PIV observations



Dynamical model

- **Location Uncertainty Model**
  `Chandramouli et al. (2018, C. Fluids)`

Data assimilation approaches

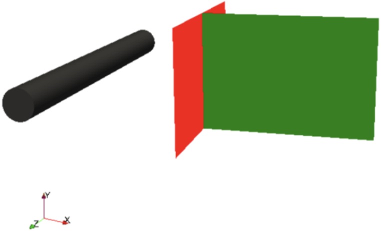
- **Incremental 4DVar approach**
  `Chandramouli` *et al.*`(2018, submitted to JCP)`
- Control inflow/outflow, initial condition and LES parameter
- Design 3D background from 2D cross PIV
  `Chandramouli` *et al.*`(2018, submitted to Exp. Fluids)`

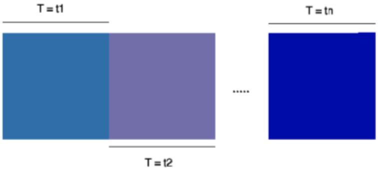
Data assimilation results

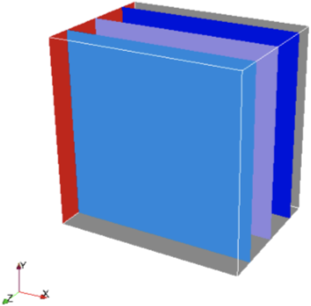
- Reconstruct 3D flow and model parameter

How to build the background in 3D?

Snapshot Optimization method (Chandramouliet *al.*, 2018)



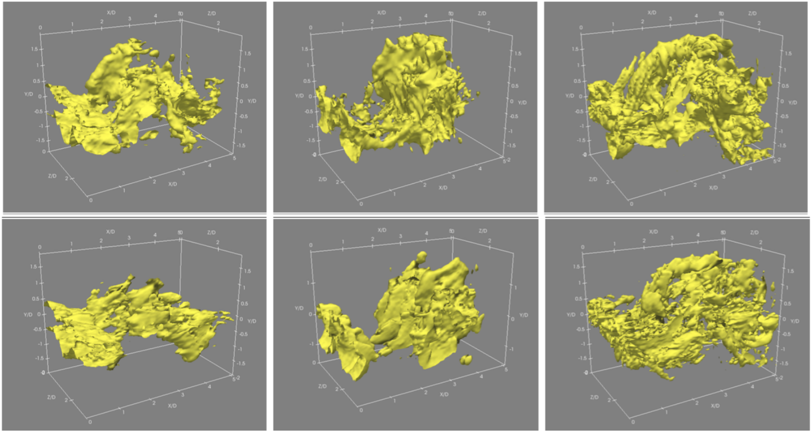




- Flow with one direction of homegeneity

How to build the background in 3D?

Snapshot Optimization method (Chandramouliet *al.*, 2018)



LES subgrid scale model

Chandramouli*et al.* (2018, C. Fluids)

Models under Location Uncertainty (MULC) :

$$\boldsymbol{u}(\boldsymbol{x}, t) = \boldsymbol{w}(\boldsymbol{x}, t) + \boldsymbol{\sigma}(\boldsymbol{x}, t) d\dot{\boldsymbol{B}}_t$$

- $\boldsymbol{u}(\boldsymbol{x}, t)$ is the instanteneous velocity field
- $\boldsymbol{w}(\boldsymbol{x}, t)$ is the large scale drift
- $\boldsymbol{\sigma}(\boldsymbol{x}, t) d\dot{\boldsymbol{B}}_t$ stands for small scales

LES subgrid scale model

Chandramouli *et al.* (2018, C. Fluids)

NS formulation as derived in Memin [2014] :

Mass conservation :

$$d_t\rho_t + \nabla\cdot(\rho\tilde{\boldsymbol{w}})dt + \nabla\rho\cdot\boldsymbol{\sigma}\, d\boldsymbol{B}_t = \frac{1}{2}\nabla\cdot(\boldsymbol{a}\nabla q)dt, \tag{6}$$

$$\tilde{\boldsymbol{w}} = \boldsymbol{w} - \frac{1}{2}\nabla\cdot\boldsymbol{a} \tag{7}$$

For an incompressible fluid :

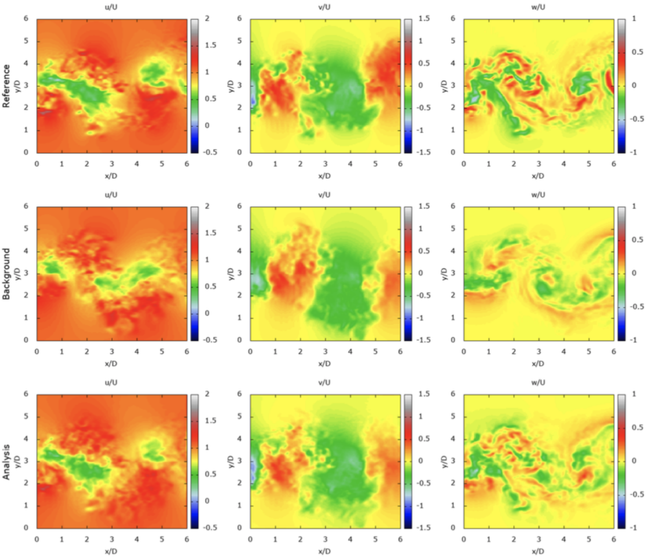
$$\nabla\cdot(\boldsymbol{\sigma} d\boldsymbol{B}_t) = 0, \quad \nabla\cdot\tilde{\boldsymbol{w}} = 0, \tag{8}$$

Momentum conservation :

$$\left(\partial_t\boldsymbol{w} + \boldsymbol{w}\nabla^T(\boldsymbol{w} - \frac{1}{2}\nabla\cdot\boldsymbol{a}) - \frac{1}{2}\sum_{ij}\partial_{x_i}(a_{ij}\partial_{x_j}\boldsymbol{w})\right)\rho = \rho\boldsymbol{g} - \nabla p + \mu\Delta\boldsymbol{w}.$$

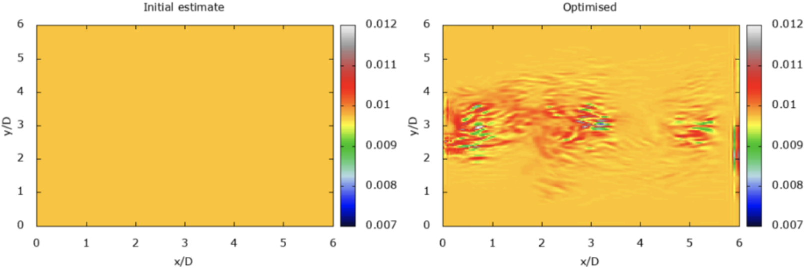
4DVar with LES subgrid scale model

3D flow reconstruction



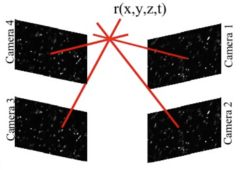
4DVar with LES subgrid scale model

Subgrid scale parameter estimation



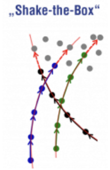
3D Particle Tracking Velocimetry: En4DVar-PTV

PIV observations



Dynamical model

- **2nd order polynome**



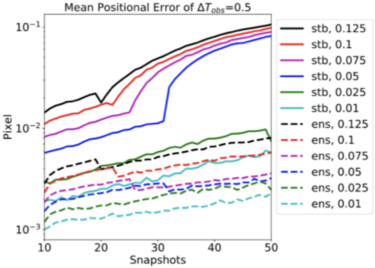
Schanz et al. (2016,EIF)

Data assimilation approaches

- **En 4DVar PTV approach** from Yang *et al.*(2018, CFTL)

Data assimilation results

- Better particle position and velocity



Sumary

- Data assimilation is a powerful technique to combine observations and models (sequential or variational) $\rightarrow$ for prediction, filtering or smoothing
- Data driven vs model driven (d vs m): when observations available $<<$ data to describe the system $\rightarrow$ model and regularization are paramount
- History of use is the search for suitable approximation that, even sub-optimal, works with non-linear, non Gaussian and high dimensional settings

Outlooks

- Dynamics model (large scale, uncertainties)
- From pseudo-observations (velocities) to observations (images)
- Control BC (inflow, outflow, ...) and model parameters (combined with IA)